Closing Tues: HW 10.1

Closing Thurs: HW 10.2

Exam 1 will be returned Tues

Entry Task (directly from HW)
Consider $P(t) = 33t + 6t^2 - t^3$.
For what value of t is $P(t)$ increasing?
(You'll need a calculator to get some decimals). Also do the full 1st deriv. number line analysis that we did in lecture on Friday.

$$P'(t) = 33 + 12t - 3t^2 \stackrel{?}{=} 0 \quad \div 3$$

$$11 + 4t - t^2 \stackrel{?}{=} 0$$

$a = -1$
$b = 4$
$c = 11$

$$t = \frac{-4 \pm \sqrt{4^2 - 4(-1)(11)}}{2(-1)} = \frac{-4 \pm \sqrt{16 + 44}}{-2} = \frac{(-4 \pm \sqrt{60})}{-2}$$

$$= -1.8730 \quad \text{or} \quad 5.8730$$

P	↘		↗		↘
P'	$-$	-1.8730	$+$	5.8730	$-$

$P'(-10) = 33 + 12(-10) - 3(-10)^2 = -387$

$P'(0) = 33 + 12(0) - 3(0)^2 = 33$

$P'(10) = 33 + 12(10) - 3(10)^2 = -147$

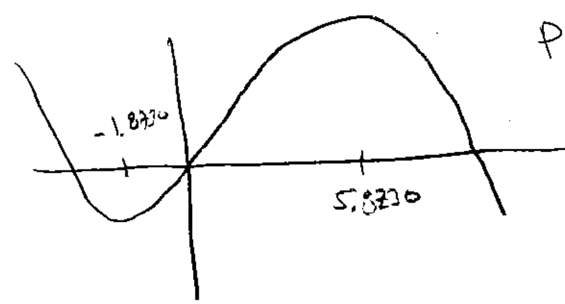
INCREASING ON

$-1.8730 < t < 5.8730$

(NOTE, IN HW IT IS AN APPLICATION WHERE ONLY POSITIVE MAKES SENSE
$0 < t < 5.8730$)

LOCAL MAX: AT $x = 5.8730$

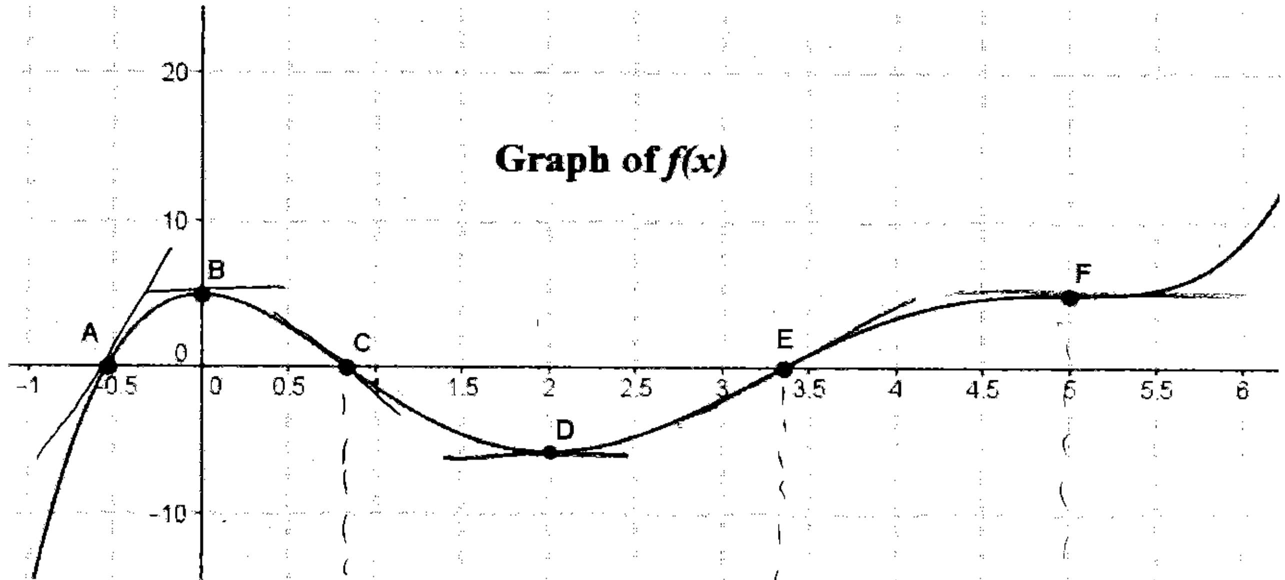
LOCAL MIN: AT $x = -1.8730$



$P(t)$

10.2 Concavity

Consider the given $y = f(x)$ graph (same graph from last lecture). Draw the tangent line at each point. Is the tangent line above or below the curve near that point?



f	CONCAVE DOWN	CONCAVE UP	CONCAVE DOWN	CONCAVE UP
f''	$-$	$+$	$-$	$+$

At $\boxed{A}$ AND $\boxed{B}$: tangent is above the graph. CONCAVE DOWN!

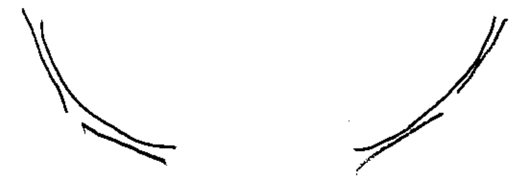
AT $\boxed{D}$: tangent is below the graph. CONCAVE UP!

AT $\boxed{C}$, $\boxed{E}$, $\boxed{F}$: tangent crosses the graph

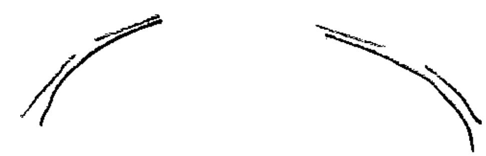
POINT OF INFLECTION.

Terminology:

If $f''(x)$ is *positive* at $x = a$,
then $f(x)$ is **concave up** at $x = a$.
This means the tangent slopes are increasing near $x = a$ and the tangent line is below the graph at $x = a$.

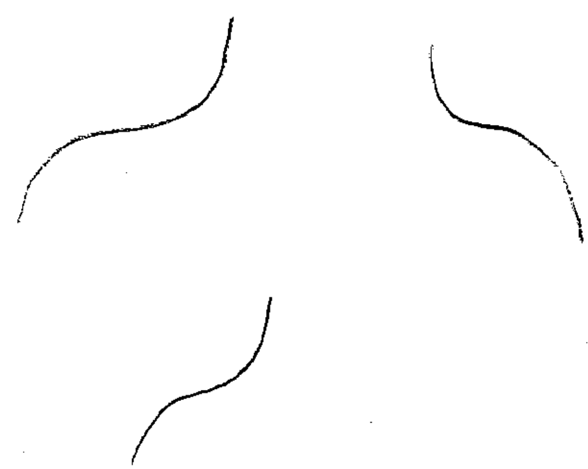


If $f''(x)$ is *negative* at $x = a$,
then $f(x)$ is **concave down** at $x = a$.
This means the tangent slopes are decreasing near $x = a$ and the tangent line is above the graph at $x = a$.



If $f''(x) = 0$ at $x = a$, then we say $x = a$ is a ***possible point of inflection***.

A ***point of inflection*** is any point where the concavity *changes*.



Example:

Let $f(x) = \frac{1}{2}x^4 - 3x^2 + 5x + 1$

Find all intervals when $f(x)$ is concave up and find all inflection points.

$$f'(x) = 2x^3 - 6x + 5$$

$$f''(x) = 6x^2 - 6 \stackrel{?}{=} 0$$

$$\Rightarrow \quad 6x^2 = 6$$

$$\Rightarrow \quad x^2 = 1$$

$$\Rightarrow \quad x = \pm 1$$

f	$\cup$		$\cap$		$\cup$
f''	$+$	-1	$-$	1	$+$

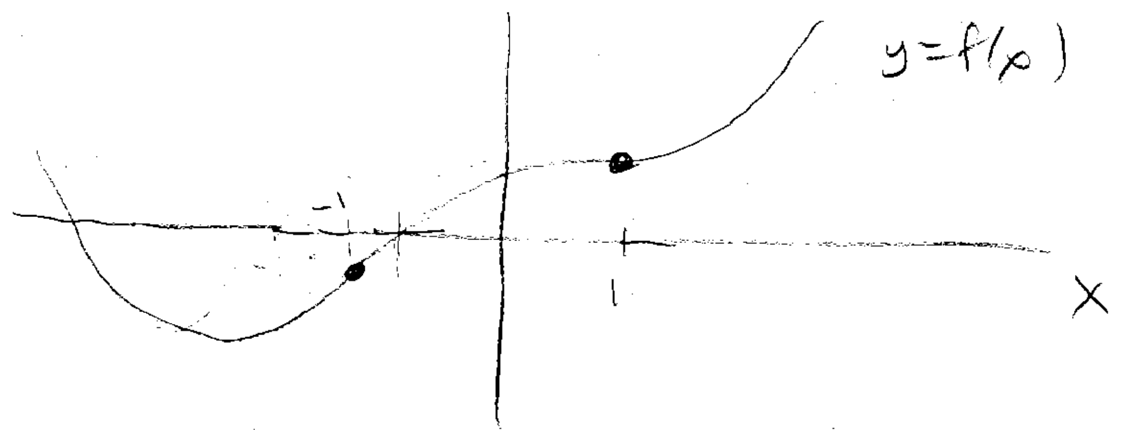
$f''(-2) = 6(-2)^2 - 6 = 24 - 6 = 18$

$f''(0) = 6(0)^2 - 6 = -6$

$f''(2) = 6(2)^2 - 6 = 24 - 6 = 18$

$\boxed{\text{CONCAVE UP: } x < -1 \text{ AND } x > 1}$

$\boxed{\text{INFLECTION PTS: } x = -1,\ x = 1}$



Summary of 1st and 2nd deriv. facts

$f(x)$	$f'(x)$	$f''(x)$
horiz. tangent	zero	
increasing	positive	
decreasing	negative	
possible inflection	hor. tangent	zero
concave up	increasing	positive
concave down	decreasing	negative

1st Deriv Analysis:(to find critical points, increasing, decreasing, local max/min, h.p.o.i)

Step 1: Critical Points

Find $f'(x)$ and solve $f'(x) = 0$.

Step 2: Draw number line. Between critical points, pick values of x and plug into $f'(x)$ to see if it is positive or negative.

Step 3: Make appropriate conclusions.

2st Deriv Analysis: (to find inflection points, concave up/down)

Step 1: Possible Inflection Points

Find $f''(x)$ and solve $f''(x) = 0$.

Step 2: Draw number line. Between possible infection points, pick values of x and plug into $f''(x)$ to see if it is positive or negative.

Step 3: Make appropriate conclusions.

Example:

Let $g(x) = x^3$.

Find all local optima and points of inflection, then sketch the graph.

1st Deriv. Analysis

$g'(x) = 3x^2 \stackrel{?}{=} 0 \Rightarrow x = 0$

For $x < 0$: $g'(-1) = 3(-1)^2 = 3$

For $x > 0$: $g'(1) = 3(1)^2 = 3$

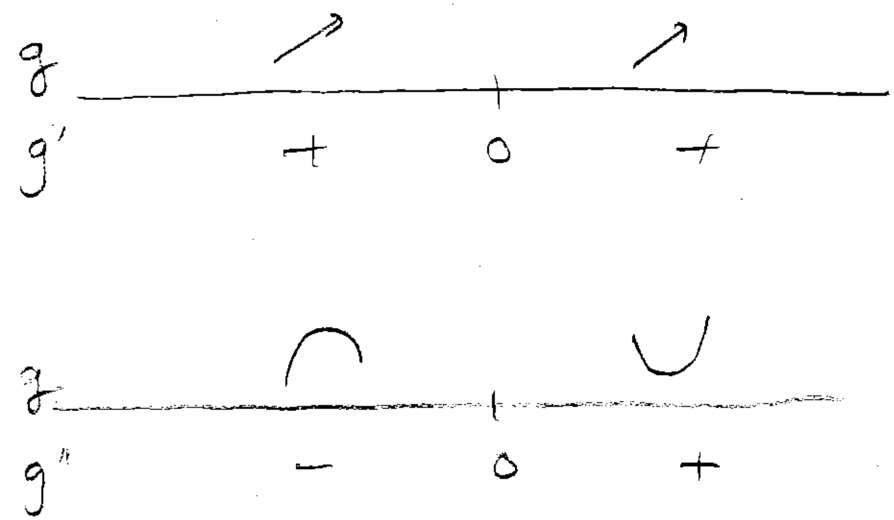
2nd Deriv. Analysis

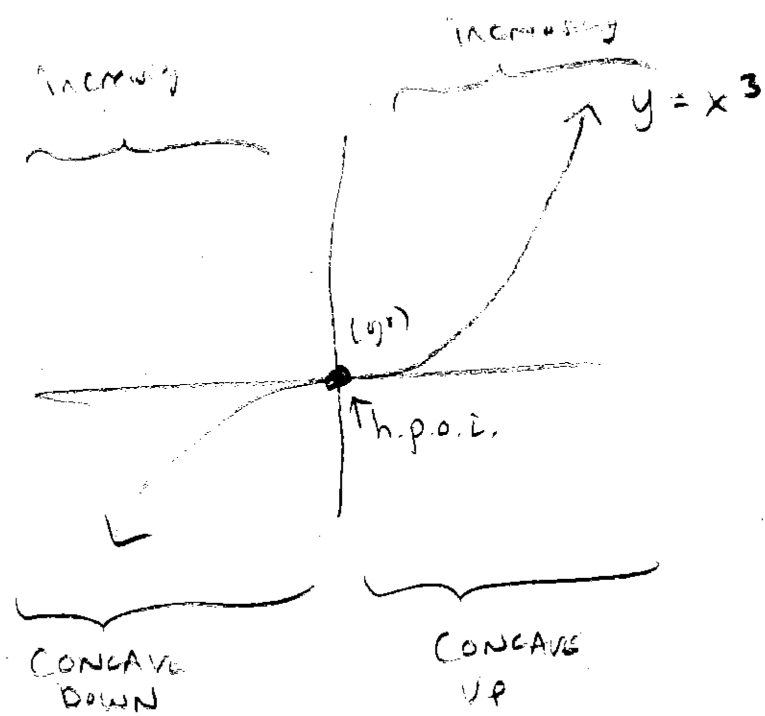
$g''(x) = 6x \stackrel{?}{=} 0 \Rightarrow x = 0$

For $x < 0$: $g''(-1) = 6(-1) = -6$

For $x > 0$: $g''(1) = 6(1) = 6$

NO LOCAL OPTIMA!





Example: Let $TC(q) = 5000q^2 + 125000$ dollars for producing q things.

Recall: Overall average cost per item is given by

$$AC(q) = \frac{TC(q)}{q} = \frac{5000q^2 + 125000}{q} \quad \$/\text{Thing}$$

Analyze AC(q).

(What does it look like?, what are relative max/min? etc….)

ONLY $q > 0$!

Simplify!

$$AC(q) = \frac{5000q^2}{q} + \frac{125000}{q}$$

$$AC(q) = 5000q + 125000q^{-1}$$

$$AC'(q) = 5000 - 125000q^{-2} \stackrel{?}{=} 0$$

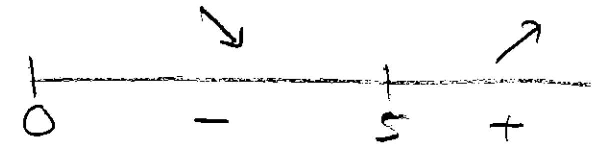
$$5000 - \frac{125000}{q^2} \stackrel{?}{=} 0$$

$\times q^2$

$$5000q^2 - 125000 \stackrel{?}{=} 0$$

$$5000q^2 = 125000$$

$$q^2 = 25 \qquad q = \pm 5$$

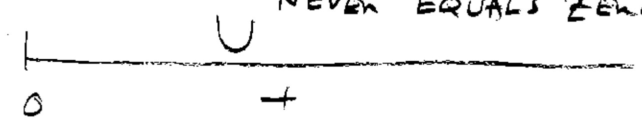


$q < 5$: $AC'(1) = 5000 - 125000 < 0$

$q > 5$: $AC'(10) = 5000 - \frac{125000}{10^2} > 0$

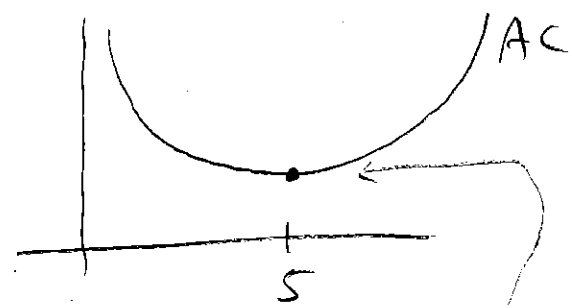
$$A''(q) = 250000q^{-3} = \frac{250000}{q^3}$$

NEVER EQUALS ZERO!



For $q > 0$: $AC''(1) = 250000 > 0$

ALWAYS CONCAVE UP.



$AC(5) = 50{,}000$ $/thing

BREAKEVEN PRICE (BEP)